QUASISTATIC AND WAVE DESCRIPTIONS OF ELASTIC BODY LOADING

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Quasistatic and wave approaches to description of the phenomenon of collision of solid bodies were developed at the end of the last century by Navier, St. Venant, and other classic scholars in the field of mechanics [1-4]. Let us recall that according to the quasistatic theory the forces developing upon collision are of a long-range character, and the colliding bodies are regarded as elastic springs, as a result of which the compressive force is a continuous function of collision time. In the wave theory the finite propagation rate of compressive forces is considered together with their subsequent transformation at the boundaries of the colliding bodies. Therefore the space–time profile of stresses, being a superposition of all the wave disturbances, departs from smoothness significantly.

Although the unity of the wave and quasistatic approaches has been demonstrated by a number of numerical studies of the collision process, a strict proof of such unity is lacking, mainly due to mathematical difficulties in precisely describing the collision process. Using the example of longitudinal collision of elastic bodies, we will perform below an approximate analysis of the wave dynamics of the collision, based on linearization of the stress profiles in each circulation of a wave through the body collided with. Simple expressions are obtained for calculating maximum collision pressures, which agree satisfactorily with the expressions of quasistatic theory and corresponding empirical results obtained for the case where the mass of the striking body is 2-3 orders of magnitude greater than the mass of the body struck.

1. We obtain the basic relationships of the quasistatic theory of collision with the example of action of a solid body (striker) of mass M on a cylindrical bar of length ℓ with planar faces of section S, mounted on a rigid anvil. Considering the bar as an elastic spring, we denote by a the amount of bar compression. Then the compressive force $F = -ka$, where k is the mechanical rigidity of the bar, equal to ES/ℓ ($E = \rho c^2$ is the modulus of longitudinal compression, ρ, c are the density and speed of sound in the material), satisfies the harmonic

$$d_{tt}F + \omega^2 F = 0, \quad F(0) = 0, \quad F(\pi/2\omega) = F_m, \quad \omega^2 = k/M, \tag{1.1}$$

whence

$$F = F_m \sin \omega t. \tag{1.2}$$

The maximum compressive force F_m can be determined from the condition of equality of the kinetic energy of the striker $Mv_0^2/2$ (where v_0 is the collision velocity) to the potential energy of bar compression $F_m^2/2k$. Introducing the pressure upon collision $p = F/S$ and the collision time, from the condition $F(t_c) = 0$, we obtain

$$p_m = (v_0/S)(Mk)^{1/2} = p_0\alpha^{-1/2}, \; t_c = \pi(M/k)^{1/2} = \pi/2\alpha^{1/2}, \; p_0 = \rho c v_0 \tag{1.3}$$

($\alpha = \rho S\ell/M$ is the ratio of striker and target masses). Note that Eq. (1.3) is written in terms of both the quasistatic and the wave approach to the collision process (in terms of the speed of sound c). The notation for p_0 coincides with the expression for pressure in the longitudinal compression wave produced at the beginning of collision. It is obvious that introduction of the force $F_0 = p_0 S$ in initial condition (1.1) has no effect on p_m.

2. Before attempting to solve the wave problem of collision, we will refine the concept of an absolutely rigid striker. To do this we find the reflection and transmission coefficients for a longitudinal planar wave of arbitrary form on the boundary (x = 0) of two bars of different sections S_1 and S_2 with parameters ρ_1, E_1 and ρ_2, E_2.

The general solution of the wave equation in bar 1 (x < 0) can be written in the form

$$u_1(x, t) = f_1(x - c_1 t) + g_1(x + c_1 t)$$

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(f_1 and g_1 are the waves incident on and reflected from the boundary). The wave entering bar 2 ($x > 0$) is described by an expression of form

$$u_2(x, t) = f_2(x - c_2 t).$$

Equating the displacements of the bar faces $u_1|_{x=0} = u_2|_{x=0}$ and the forces acting on each bar,

$$S_1 E_1 \partial_x u_1|_{x=0} = S_2 E_2 \partial_x u_2|_{x=0},$$

we find $(1 - z_2/z_1) f_1(-\zeta) = (1 + z_2/z_1) g_1(\zeta)$, where $z_i = \rho_i c_i S_i$ ($i = 1, 2$) are the shock impedances of the bars, $\zeta = c_1 t$. Thus,

$$u(x, t) = f_1(x - \zeta) + (z_1 - z_2)(z_1 + z_2)^{-1} f_1(-x - \zeta),$$

i.e., the reflection coefficient $\varphi = (z_1 - z_2)(z_1 + z_2)^{-1}$ and the transmission coefficient $\psi = 1 + \varphi = 2z_1(z_1 + z_2)^{-1}$. If the bar materials are identical ($\rho_1 c_1 = \rho_2 c_2$) and $S_1 \gg S_2$, then $\varphi = 1$ and $\psi = 2$. In accordance with the analysis of [5], in this case bar 1 may be considered absolutely rigid relative to bar 2.

We will consider longitudinal collision of an absolutely rigid body on an elastic bar with fixed end at the point $x = \ell$. At the moment of collision ($t = 0$) the mass velocity at the free end of the bar ($x = 0$) is equal to v_0, and the initial compressive stress $p_0 = \rho c v_0 > 0$. Displacements of bar particles are described by a one-dimensional wave equation with d'Alembert's solution

$$u(x, t) = u_+(t - x/c) + u_-(t + x/c)$$

(u_+ are displacements produced by all waves moving from the struck end, while u_- are from waves arriving at the struck end of the bar).

Before the time of arrival of the wave at the fixed end the problem solution is defined by the single term

$$u(x, t) = u_0(t - x/c), \quad 0 \leqslant t \leqslant l/c. \tag{2.1}$$

Writing the condition at the free end in the form

$$M\partial_{tt} u = M\partial_t v = -S\sigma, \quad \sigma = -E\partial_x u, \quad v = \partial_t u \tag{2.2}$$

(σ, v are particle stress and velocity in the section $x = 0$) and using Eq. (2.1), we obtain an equation for the change in compressive stress with time:

$$d_t \sigma + 2\alpha\sigma/T = 0, \quad \sigma(0) = p_0, \quad T = 2l/c,$$

solution of which yields

$$\sigma = p_0 \exp(-2\alpha t/T), \quad 0 \leqslant t \leqslant T. \tag{2.3}$$

Thus, because of the dynamic resistance of the bar the striker velocity and pressure in the bar decrease exponentially and a planar compression wave with decreasing stress profile begins to propagate along the bar.

After reflection of the wave from the fixed end the solution takes the form

$$u(x, t) = u_0(t - x/c) + u_0'(t + x/c), \quad 0 \leqslant t \leqslant T. \tag{2.4}$$

At the point $x = \ell$ we have the condition $u(\ell, t) = 0$, which leads to the equality

$$u_0'(\xi) = -u_0(\xi - T), \qquad \xi = t + l/c.$$

The latter condition is satisfied for any positive ξ value, so that solution (2.4) may be written as

$$u(x, t) = u_0(t - x/c) - u_0(t + x/c - T).$$

Hence it follows, in particular, that the stress at the fixed end

$$\sigma(l, t) = -E\partial_x u|_{x=l} = \sigma_0(\xi) + \sigma_0(\xi - T) = 2\sigma_0(\xi)$$

is double the stress in the forward wave.

From the beginning of the second circulation of the wave along the bar particle displacements are determined by two waves moving from the struck end, and a single wave propagating toward that end:

$$u(x, t) = u_0(t - x/c) - u_0(t + x/c - T) + u_1'(t - x/c) = u_1(t - x/c) - u_0(t + x/c - T). \tag{2.5}$$

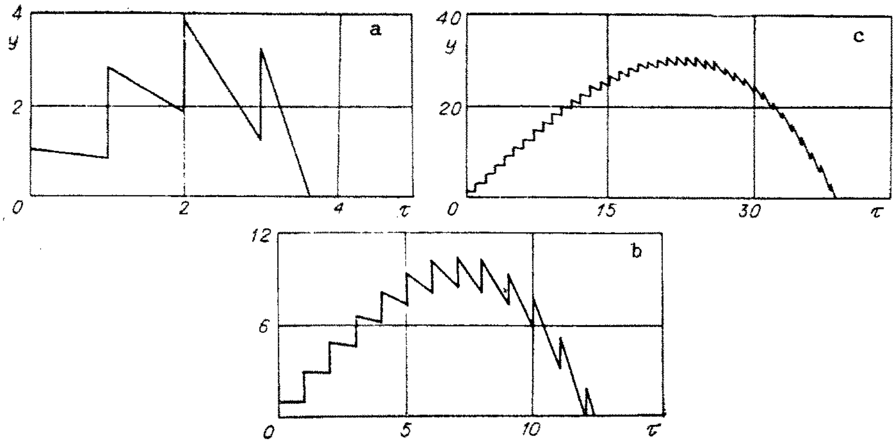

Fig. 1

With the aid of Eq. (2.5) we obtain an expression for stresses and particle velocities at the point x = 0:

$$\sigma = -E\partial_x u|_{x=0} = \sigma_1(t) + \sigma_0(t-T),$$
$$v = \partial_t u|_{x=0} = (\rho c)^{-1}(\sigma_1(t) - \sigma_0(t-T)), \quad \sigma_0 = p_0 \exp(-2\alpha t/T). \tag{2.6}$$

We denote by $\sigma_n(t)$ the total compressive stresses from all waves appearing at the free end of the bar at times nT, and by $\sigma_{n-1}(t - T)$, stresses from waves appearing at previous moments (n − 1)T and delayed by a time T due to circulation along the bar. Then, in analogy to Eq. (2.6) we have [1]

$$\sigma(t) = \sigma_n(t) + \sigma_{n-1}(t-T), \quad v(t) = (\rho c)^{-1}(\sigma_n(t) - \sigma_{n-1}(t-T)). \tag{2.7}$$

Substitution of Eq. (2.7) in the striker braking law (2.2) permits establishment of a relationship between $\sigma_n(t)$ and $\sigma_{n-1}(t)$:

$$\sigma_n(t) = \sigma_{n-1}(t-T) - (4\alpha/T)\exp(-2\alpha t/T)\left[\int \exp(2\alpha t/T)\,\sigma_{n-1}(t-T)\,dt + C\right]. \tag{2.8}$$

We find the integration constant C from the condition that at times nT the compressive stresses at the contact with the striker increases by $2p_0$:

$$\sigma_n(nT) + \sigma_{n-1}((n-1)T) = \sigma_{n-1}(nT) + \sigma_{n-2}((n-1)T) + 2p_0. \tag{2.9}$$

Using the initial value $\sigma_0(t)$ given by Eq. (2.6), [1] found the first several values of $\sigma_n(t)$. For larger n values the expressions for $\sigma_n(t)$ become cumbersome and difficult to evaluate. Also presented there were results of numerical $\sigma(t)$ calculations for several collision cases.

At the same time, by using Eqs. (2.8), (2.9) simple analytical expressions can be obtained for the stresses $\sigma_n(t)$ linearized for $nT \leqslant t \leqslant (n+1)T$. In this case with no loss of generality we can determine the complete pattern of the collision, for which it is sufficient to take $\alpha \leqslant 1$.

Representing $\sigma_0(t)$ in dimensionless form

$$0 \leqslant \tau \leqslant 1: \; y_0(\tau) = 1 - 2\alpha\tau, \quad \tau = t/T, \quad y(\tau) = \sigma(\tau)/p_0,$$

we find the first several values of $y_n = \sigma_n/p_0$:

$$1 \leqslant \tau \leqslant 2: \; y_1(\tau) = y_0(\tau) + 1 + 6\alpha(1-\tau),$$
$$2 \leqslant \tau \leqslant 3: \; y_2(\tau) = y_1(\tau) + 1 + 10\alpha(2-\tau),$$
$$3 \leqslant \tau \leqslant 4: \; y_3(\tau) = y_2(\tau) + 1 + 14\alpha(3-\tau),$$

whence we obtain a general expression for $y_n(\tau)$

$$n \leqslant \tau \leqslant n+1: \; y_n(\tau) = y_{n-1}(\tau) + 1 + 2(2n+1)\alpha(n-\tau) \tag{2.10}$$

and the compressive stress at the free face $n \leqslant \tau \leqslant n+1$

$$y(\tau) = 2n + 1 + (4/3)n(n+1)(2n+1)\alpha - 2(1 + 2n(n+1))\alpha\tau. \tag{2.11}$$

Considering times τ not too small, for qualitative study of Eq. (2.11) we can set n = τ and define

TABLE 1

Parameter	Equation	α		
		10^{-1}	10^{-2}	10^{-3}
y_m	(2.11)	3,800	10,38	30,79
	(2.13)	3,834	10,38	30,80
	(1.3)	3,162	10,00	31,62
τ_c	(2.11)	3,640	12,30	38,64
	(2.13)	3,808	12,22	38,72
	(1.3)	4,967	15,71	49,67

$$y(\tau)=1+2\tau(1-(\alpha/3)(1+2\tau^2)). \tag{2.12}$$

With the aid of Eq. (2.12) we find the times at which the maximum stress is reached $[d_\tau y(\tau_m) = 0]$, the duration of the collision $[y(\tau_c) = 0]$, and the maximum stress during collision $y_m(\tau_m)$:

$$\tau_m=((3-\alpha)/6\alpha)^{1/2},\quad \tau_c\simeq\sqrt{3}\,\tau_m,$$
$$y_m=1+(4/9)(3-\alpha)\tau_m\simeq 1+0{,}9428\alpha^{-1/2}. \tag{2.13}$$

As would be expected, Eqs. (1.3) and (2.13) for the quasistatic and wave theories of collision practically coincide for $\alpha \ll 1$. The expression for the maximum pressure y_m is close to the experimentally determined $y_m = 1 + \alpha^{-1/2}$ [1].

Figure 1 shows functions $y(\tau)$, calculated with Eq. (2.11) for $\alpha = 10^{-1}$, 10^{-2}, and 10^{-3} (a-c). It is evident that for integral τ the stress changes discontinuously. With decrease in α the function $y(\tau)$ approaches ever more closely the smooth curve specified by quasistatic theory ($y \sim \sin\tau$).

Table 1 presents results of calculations of several collision parameters using the wave theory of Eq. (2.11), the approximate relationships of Eq. (2.13), and the quasistatic theory of Eq. (1.3).

In accordance with Eq. (2.7) the functions $y_n(\tau)$ can be used to define the stresses in any other section of the bar. Since the maximum stress at its fixed end y_{max} is equal to double the maximum value $y_n(\tau_{max})$, by writing Eq. (2.10) in the form

$$y_n(\tau)=n+1+\alpha(n(n+1)(4n+5)/3-2(n+1)^2\tau) \tag{2.14}$$

and taking $n = \tau$, we find

$$y_n(\tau)=(\tau+1)(1-(\alpha\tau/3)(2\tau+1)),$$

whence

$$\tau_{max}=(1/2\alpha+1/12)^{1/2}-1/2\simeq(2\alpha)^{-1/2}\simeq\tau_m,$$
$$y_{max}=2y_n(\tau_{max})\simeq 1+(2/3)(2\alpha)^{1/2}\simeq y_m,$$
$$\tau_* = ((24/\alpha+1)^{1/2}-1)/4\simeq(3/2\alpha)^{1/2}\simeq\tau_c \tag{2.15}$$

(with approximations for $\alpha \ll 1$). Table 2 presents calculated values of collision parameters on the fixed bar end.

Comparing the various calculation results in Tables 1 and 2, we note that the collision time at the free end τ_c is greater than at the fixed end τ_*, by approximately a semiperiod of the wave circulating along the bar. In correspondence with the law of conservation of momentum, the maximum compressive stress at the fixed end y_{max} is greater than at the free end y_m, with the difference becoming insignificant as $\alpha \to 0$.

We will now consider the change in striker velocity. An expression for $v(t)$ can be found from Eqs. (2.7), (2.10), although it will contain significant uncertainty due to accumulation of errors resulting from subtracting small quantities. Therefore, having substituted Eq. (2.12) into the equation of motion (2.2) and integrating the latter with the condition $v(0) = v_0$, we obtain

$$V(\tau)=v/v_0=1-2\alpha\tau(1+(1-\alpha/3)\tau-\alpha\tau^3/3).$$

At the moment when maximum stress is reached, the velocity

$$V(\tau_m)=1/6+5\alpha/9-(2\alpha(1-\alpha/3))^{1/2}$$

is close to zero, and at the end of the collision

$$V(\tau_c)=-1/2+\alpha-(2\alpha(3-\alpha))^{1/2}<0.$$

TABLE 2

Parameter	Equation	α		
		10^{-1}	10^{-2}	10^{-3}
y_{max}	(2.14) (2.15)	4,000 4,056	10,40 10.45	30,82 30,82
τ_*	(2.14) (2.15)	3,375 3,631	11,65 12,00	38,32 38,48

For change in α from $5\cdot 10^{-2}$ to $5\cdot 10^{-4}$ the velocity reestablishment coefficient for the collision $\varepsilon = -V(\tau_c)$ decreases from 0.993 to 0.554, i.e., within the limits of values typical of laboratory pile driver collision experiments [2, 4].

We will note that the results of the ε calculations should not be overrated, since they were obtained, first, within the framework of a linearized model of wave disturbances, and second, under conditions difficult to obtain in practice (plane-parallel approach of bar and striker faces, absence of wave dispersion, etc.). Moreover, the assumption of an absolutely rigid striker eliminates dependence of ε on the material of the colliding bodies.

Thus, the above analysis of wave and quasistatic descriptions of the process of longitudinal collision of elastic bodies has shown that at small ratios of bar/striker mass both approaches yield similar results as regards maximum stresses and collision times, which agree well with data from the literature. The values found for the velocity reestablishment coefficient upon collision do not contradict experimental values.

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NUMERICAL MODELING OF PETROLEUM HEATING AND FILTRATION IN A PLATE UNDER THE ACTION OF HIGH-FREQUENCY ELECTROMAGNETIC RADIATION

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The use of high-frequency electromagnetic radiation is a promising method for intensifying production of high viscosity petroleum. Because of its deep penetration and consequent volume heat liberation, electromagnetic radiation enables a much higher and more uniform heating rate and a higher efficiency than the traditional thermal methods of heated vapor or hot liquid. However, realization of such capabilities requires detailed study of the heat-mass transport processes which occur, in order to discover optimal operating regimes. Estimates of penetration depth, temperature distribution, and filtration rate in one-dimensional models were made in [1-4]. A two-dimensional plate model was studied in [5], but without consideration of petroleum filtration (plate heating with closed well). At the same time it is possible (and field tests have been carried out [6]) to heat a plate electromagnetically while simultaneously extracting oil, to model which consideration of both heat transport and petroleum filtration in the porous plate are obviously necessary. Determination of optimum parameters in such a regime is the goal of the present study.

Model and Equations. Numerical studies were performed with a two-dimensional axisymmetric model, a diagram of which is shown in Fig. 1. The petroleum stratum is contained

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